A visual examination of the hot wire signals clearly revealed the highly intermittent character of the wake turbulence. At the probe locations chosen, the intermittency factor was about 0.5 and the average frequency of occurrence of the turbulent bursts corresponded to the frequency of the peak in the power spectrum. The photograph of concurrent hot wire signals from two probes at $\theta = 180°$, shown in Fig. 4, illustrates the tendency of the intermittent bursts to be antiphase, at this value of θ. This behavior is reflected by the high negative coherence found at $\theta = 180°$ and $S = 0.135$, as shown in Fig. 3, and is consistent with the notion of a simple "flapping" motion of the turbulent core, the core tending to engulf the hot wire probes alternately. The coherence results for low frequencies, shown in Fig.3, are consistent wih a slow, random variation in the orientation of such a flapping motion.

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Elastic Wave Surfaces in Heterogeneous Anisotropic Plates

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Introduction

ELASTIC wave surfaces due to impact on anisotropic plates have recently been investigated by Moon.[1] In the analysis, Moon employed an effective modulus plate theory and considered a class of specially laminated fiber-reinforced composite plates, which uncouples the transverse, bending and extensional displacements; a wave surface approach was used to describe the propagation of plane acceleration waves. The present Note is concerned with elastic wave surfaces in plates that are composed of layers possessing arbitrary anisotropy. In this case a severe coupling exists among the shear bending, twisting and extensional effects, resulting in simultaneously coupled wave surfaces in the plane of the plate. We follow a general laminated plate theory[2] and apply a control volume approach for the analysis.[3]

Explicit solutions for the coupled wave surfaces and their velocities are obtained. Several numerical problems involving laminated fiber-reinforced composite plates are presented and their unique features discussed.

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Analysis

Let us consider a thin laminated plate of thickness h (Fig. 1). The laminae comprising the plate are assumed to be individually homogeneous and anisotropic. Thus the inhomogeneity of the plate occurs only in the thickness direction. We shall consider a plane wave front which originates at an arbitrary point in the plate, for convenience let us say at the origin of the (x, y, z) system, and propagates in the x, y plane. At any given instant, the wave surface is denoted by S. Let $\bar{n}$ be the normal of S at a point A on S, and let $\bar{s}$ be the tangent of S at the same point. The wave surface S is assumed to propagate in the direction $\bar{n}$ with a constant speed $c(\bar{n})$.

Let u, v, w, ψ_n and ψ_s be the displacements, referred to the local coordinates (n, s, z). Following the laminated plate theory,[2] their forms are assumed as

$$u = u^o(n, s, t) + z\psi_n(n, s, t), \quad v = v^o(n, s, t) + z\psi_s(n, s, t), \quad w = w^o(n, s, t) \tag{1}$$

The kinematic (continuity) conditions across S at any given point on S require (see Ref. 3)

$$[u^o] = [v^o] = [w^o] = [\psi_n] = [\psi_s] = 0$$
$$[u_{,n}{}^o, v_{,n}{}^o, w_{,n}{}^o, \psi_{n,n}, \psi_{s,n}] = (1/c)[u_{,t}{}^o, v_{,t}{}^o, w_{,t}{}^o, \psi_{n,t}, \psi_{s,t}]$$
$$(u, v, w, \psi_n, \psi_s)_{,s} = 0$$

where [ ] represent a discontinuity of the enclosed quantity across S.

With these conditions, the plate constitutive relations, when referred to local coordinates (n, s, z) yield

$$\begin{bmatrix} N_n \\ N_{ns} \\ M_n \\ M_{ns} \\ Q_n \end{bmatrix} = \begin{bmatrix} A_{11} & A_{16} & B_{11} & B_{16} & A_{15} \\ A_{16} & A_{66} & B_{16} & B_{66} & A_{56} \\ B_{11} & B_{16} & D_{11} & D_{16} & B_{15} \\ B_{16} & B_{66} & D_{16} & D_{66} & B_{56} \\ A_{15} & A_{56} & B_{15} & B_{56} & A_{55} \end{bmatrix} \begin{bmatrix} u_{,n}{}^o \\ v_{,n}{}^o \\ \psi_{n,n} \\ \psi_{s,n} \\ w_{,n}{}^o \end{bmatrix} \tag{2}$$

where N, Q, M and the constants A, B and D,s take the same meaning as those defined in Ref. 2 only in this case all quantities are referred to the local coordinates (n, s, z).

The dynamic relations across the wave surface are established by defining a control volume which is located on S at point A, as shown in Fig. 1. Since the control volume moves with the wave front, an observer fixed with it, sees a normal influx of mass entering with a speed $U_1 = c - u_{1,t}$, and a normal efflux of mass leaving with a speed $U_2 = c - u_{2,t}$. Thus the steady-state conservation of mass for the control volume yields

$$\int (\rho_2 U_2 - \rho_1 U_1)\, dz = 0 \tag{3}$$

where ρ is the mass density of the material and the subscripts 1 and 2 refer to the properties ahead of and behind the wave front S, respectively. The integral is carried over the thickness of the plate $[(-h/2), (h/2)]$.

It is noted that condition (3) is satisfied by a more restrictive condition, resulting from the classical thin plate assumptions, namely $\rho_2 U_2 = \rho_1 U_1$. The force and moment resultants acting upon the control volume must satisfy the equations of balanced momenta. These are

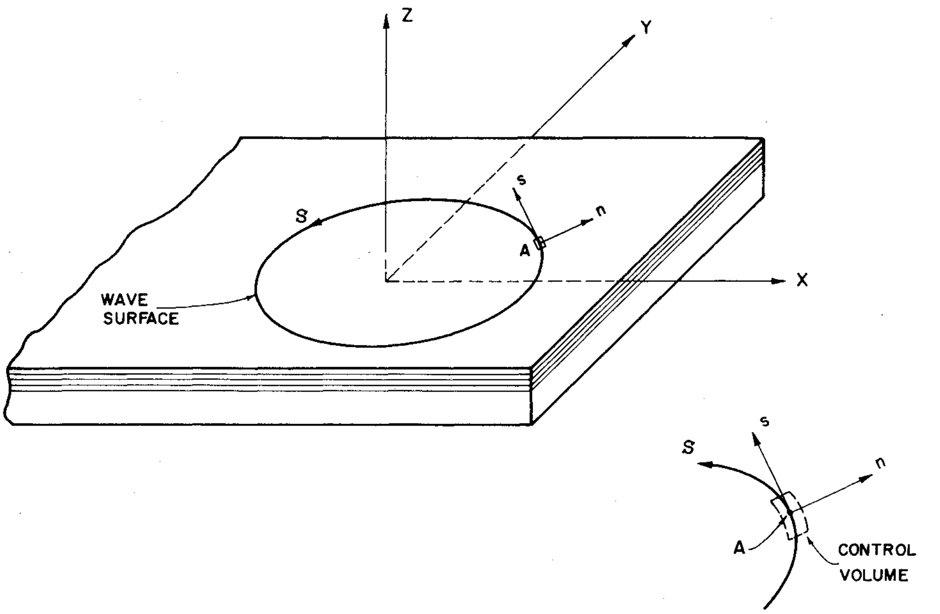

Fig. 1 Geometry of the plate, the wave surface, and the control volume.

$$\left.\begin{matrix}[N_n]\\ [M_n]\end{matrix}\right\} = \int \rho_1 U_1 (U_2 - U_1) \begin{Bmatrix}1\\ z\end{Bmatrix} dz = \begin{Bmatrix}P\\ R\end{Bmatrix} c^2[u_{,n}{}^o] + \begin{Bmatrix}R\\ I\end{Bmatrix} c^2[\psi_{n,n}] \tag{4}$$

$$\left.\begin{matrix}[N_{ns}]\\ [M_{ns}]\end{matrix}\right\} = \int \rho_1 U_1 (v_2 - v_1) \begin{Bmatrix}1\\ z\end{Bmatrix} dz = \begin{Bmatrix}P\\ R\end{Bmatrix} c^2[v_{,n}{}^o] + \begin{Bmatrix}R\\ I\end{Bmatrix} c^2[\psi_{s,n}] \tag{5}$$

$$[Q_n] = \int \rho_1 U_1 (w_2{}^o - w_1{}^o) dz = Pc^2[w_{,n}{}^o] \tag{6}$$

In obtaining these relations, we have retained only the linear terms of the displacements, consistent with the plate theory. In addition, we have assumed the density a function of z alone in presenting the following quantities

$$(P, R, I) = \int \rho_1 (1, z, z^2) dz \cong \int \rho_0 (1, z, z^2) dz \tag{7}$$

where ρ_0 is the undisturbed density of the laminae.

Using the relations (3–7), we obtain a system of five linear algebraic equations relating the discontinuities in the normal derivatives of u^o, v^o, w^o, ψ_n and ψ_n and ψ_s as follows

$$\begin{bmatrix} A_{11}-Pc^2 & A_{16} & B_{11}-Rc^2 & B_{16} & A_{15} \\ A_{16} & A_{66}-Pc^2 & B_{16} & B_{66}-Rc^2 & A_{56} \\ B_{11}-Rc^2 & B_{16} & D_{11}-Ic^2 & D_{16} & B_{15} \\ B_{16} & B_{66}-Rc^2 & D_{16} & D_{66}-Ic^2 & B_{56} \\ A_{15} & A_{56} & B_{15} & B_{56} & A_{55}-Pc^2 \end{bmatrix} \begin{bmatrix} [u_{,n}{}^o] \\ [v_{,n}{}^o] \\ [\psi_{n,n}] \\ [\psi_{s,n}] \\ [w_{,n}{}^o] \end{bmatrix} = 0 \tag{8}$$

In order for a nontrivial solution to exist, the determinant of the coefficient matrix of Eqs. (8) must vanish. Thus, five possible wave front speeds, c, may be determined for any given direction $\bar{n}$.

Since the vanishing determinant represents a fifth-order equation in c^2, numerical, rather than analytical, techniques must be used to obtain a solution. However, it is of interest to note that, if the laminated plate has, for each lamina, a monoclinic symmetry (i.e., a plane symmetry with respect to the midplane of the lamina) the constants $A_{15} = A_{56} = B_{15} = B_{56} = 0$ for all directions in the x, y plane. In such a case, $[w_{,n}{}^o]$ is uncoupled from the system. Consequently, $c_5{}^2 = A_{55}/P$ where c_5 represents the propagation speed of the discontinuity $[w_{,n}{}^o]$ in the direction n.

Furthermore, if, in addition, the plate has a symmetry with respect to the x, y plane, the bending-extensional couplings B_{ij} become zero for all i and j. Then, Eqs. (8) separate further yielding two quadratic equations whose roots are

$$c_{1,2}{}^2 = \frac{(A_{11}+A_{66}) \pm [(A_{11}-A_{66})^2 + 4A_{16}{}^2]^{1/2}}{2P} \tag{9}$$

and

$$c_{3,4}{}^2 = \frac{(D_{11}+D_{66}) \pm [(D_{11}-D_{66})^2 + 4D_{16}{}^2]^{1/2}}{2I} \tag{10}$$

In the particular case, such as that considered by Moon,[1] the extensional and the bending rigidities are proportional, i.e.,

$$A_{ij}/D_{ij} = P/I = \text{const}, \quad i,j = 1,2,6$$

Equations (9) and (10) are identical causing $c_{1,2}$ and $c_{3,4}$ to coincide [see Eqs. (17) and (23), Ref. 1].

Numerical Illustrations

For the numerical illustrations, plates which are laminated with unidirectional fiber-reinforced composite layers are con-

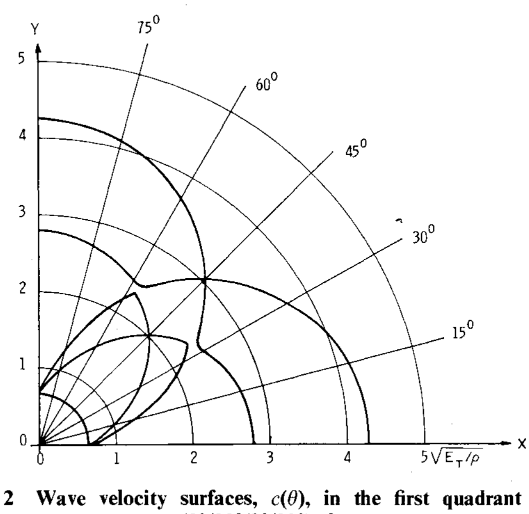

Fig. 2 Wave velocity surfaces, $c(\theta)$, in the first quadrant of the (0°/90°/0°/90°) plate.

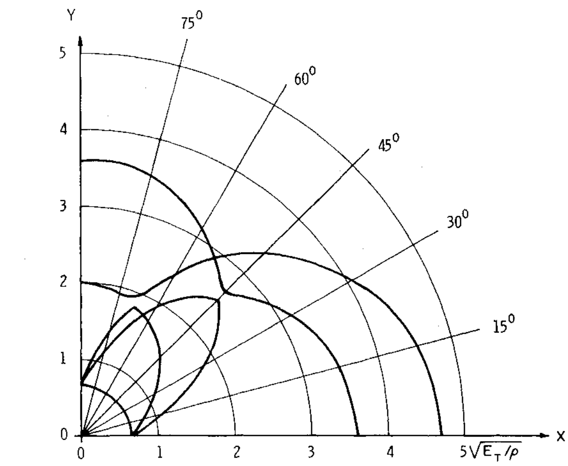

Fig. 3 Wave velocity surfaces, $c(\theta)$, in the first quadrant of the (0°/90°/90°/0°) plate.

sidered. The material properties of these layers are described by the following engineering constants‡:

$$E_L = 25 \times 10^6 \text{ psi}, \quad E_T = 10^6 \text{ psi}, \quad G_{LT} = 0.5 \times 10^6 \text{ psi}$$
$$\nu_{LT} = 0.25, \quad \nu_{TT} = 0.35, \quad \rho_0 = 0.073 \text{ lb/in.}^3$$

where E is Young's modulus, G is the shear modulus, ν is Poisson's ratio, and the subscripts L and T indicate directions parallel and normal to the fibers, respectively.

We have considered two plates each of which is made of four layers having different lay-up angles and/or lamination sequences, namely: a) (0°/90°/0°/90°); b) (0°/90°/90°/0°), where the angles are positive when measured counterclockwise from the x axis to the fiber direction.

The roots of the vanishing determinant of Eqs. (8) are determined by the Newton-Raphson technique. In general, five distinct roots representing five possible wave speeds exist in any given direction. However, in certain preferred directions, the material properties are such that repeated roots may exist.

Figures 2 and 3 show the wave velocity surfaces in the first quadrant of the x, y plane, for (0°/90°/0°/90°), and (0°/90°/90°/0°) laminates, respectively. The velocity is nondimensionalized by a factor of $(E_T/\rho_0)^{1/2}$. It is pointed out that the slowest velocity surface, which is associated with the transverse displacement w, is uncoupled from the other four surfaces, since the material is monoclinic. However, all the other four velocity surfaces may be severely coupled. On each of the corresponding wave surfaces, discontinuities in normal forces, shear forces and bending and twisting moments exist simultaneously. Their relative magnitudes may be determined from Eqs. (8) once the wave speeds c have been determined.

Multiple coupled one-dimensional stress waves in a heterogeneous plate were first treated by Wang, Chou, and Rose[6] using the method of characteristics. Recent experiments conducted at Drexel Univ. using a low-frequency ultrasonic transducer to produce normal-to-the-plate pulses show only two distinct wave groups traveling in the plane of the plate. Clearly, a definitive conclusion cannot be drawn at least presently, due to the limited data available.

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‡ These values are typical of high modulus graphite-epoxy composites. Such material layers may be considered as being square symmetric. The computation for the plates' rigidities and their transformation to local coordinates was carried out following the outlines in Refs. 4 and 5.

Three-Dimensional Separation for Interaction of Shock Waves with Turbulent Boundary Layers

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TWO-DIMENSIONAL shock boundary-layer interaction has been studied extensively and the flow mechanism as well as separation criteria are now fairly well-known. However, flow separation on practical configurations (e.g., corner regions such as wing-body junctions, rectangular inlet modules) is often three-dimensional. Therefore, it is necessary to determine when separation occurs in three-dimensional flows, means of predicting such separation, and the effects of this separation on static and total pressure in the flowfield as well as surface heat transfer. This Note presents experimental turbulent three-dimensinal separation results at $M_\infty = 5.9$ and correlations with existing three-dimensional as well as two-dimensional separation data.

This study was made with the streamwise corner model sketched in Fig. 1a at a freestream Reynolds number of

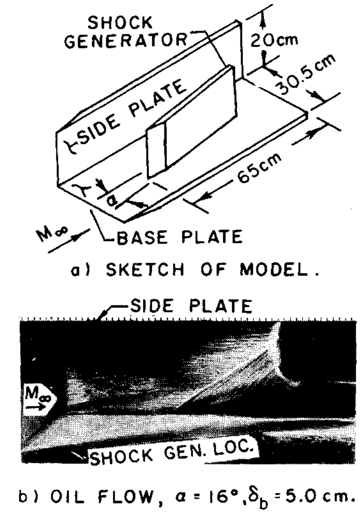

Fig. 1 Streamwise corner model.

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